

THE ARITHMETIC OF LADY BUGS

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**THIS GAME CAN BE FOUND AT JRMF.ORG
IT IS ALSO SOMETIMES KNOWN AS THE BUBBLING
CAULDRONS.**




SCHUR'S THEOREM (ISSAI SCHUR)

For any k leaves ($k > 1$), there exists a number $n > 3$ of lady bugs numbered 1 to n such that no matter how they are placed on the leaves, there will be a leaf where a lady bug carries a number that is the sum of two numbered lady bugs on that same leaf.

ONE CAN CHOOSE THE SEQUENCE OF NUMBERS

- use only the odd numbered lady bugs**
- use only the even lady bugs**
- use only the non multiples of 3**
- use the Fibonacci sequence**
- choose your favorite sequence**

2 Leaves:

Non Multiples of _____	Maximum Number That Can be Placed
TWO	Infinite
THREE	Infinite
FOUR	Infinite
FIVE	Infinite
SIX	Finite
SEVEN	Finite
EIGHT	Finite 

ONE CAN CHANGE THE NUMBER OF LEAVES

And then choose your favorite sequence

Non Multiples of _____	Maximum Number That Can be Placed
TWO	Infinite
THREE	Infinite
FOUR	Infinite
FIVE	Infinite
SIX	Infinite
SEVEN	Infinite
EIGHT	Infinite
NINE	Infinite
TEN	Infinite
ELEVEN	Infinite

ENOUGH COLLABORATION! YOU CAN TURN THE GAME INTO A DUEL

The first one makes all the lady bugs fly away loses the game.

- will someone ever win the game?**
- is there a winning strategy?**
- what is the fastest way to win?**

CHOOSE YOUR GAME AND HAVE FUN!

Be sure to check out <https://jrmf.org/> for more fun activities.

You can find useful resources on Math Circles (including Math Teachers Circles) at <https://mathcircles.org/>

And if you have any questions, or are interested in starting a math circle at your institution or in your local community, please reach out at:

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